Université Paris-Est Marne-la-Vallée
Université Paris-Est Créteil

École des Ponts ParisTech
Université d'Évry Val d'Essonne

Master's degree
MATHEMATICS AND APPLICATIONS
Academic year 2011-2012

This master's degree in "Mathematics and applications" is a research oriented graduate program. It is jointly organized by Université Paris-Est Marne-la-Vallée, École des Ponts ParisTech, Université Paris-Est Créteil, Université Évry Val d'Essonne. This program is connected with the doctoral schools *Mathématiques et STIC (MSTIC)*, within Université Paris-Est, and *Sciences et Ingénierie*, hosted by the University of Évry.

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Presentation of the degree

The master's degree "Mathematics and Applications" is a two-year graduate program. Students with a good background in mathematics (corresponding to four years in college) may be admitted in the second year. The following is a description of the second year program.

Students may select their second year courses or groups of courses (UE = "Unité d'Enseignement", which may consist of one or more courses) according to the following orientations (the UE numbers refer to the list of courses on page 4):

Finance This orientation focuses on the modeling of financial markets and numerical methods and relies on the final year of the engineering program at École Des Ponts. It is limited to about twenty students. The program is made of one group of compulsory courses (UE "tronc commun finance" F1) and one group of optional courses (UE "Mathématiques financières approfondies" F2).

The Mathfi project, a research group in association between Ecole des Ponts ParisTech, INRIA (Institut National de Recherche en Informatique et Automatique) and UPEMLV, is responsible for the scientific supervision of the program. This group develops a software for option pricing in cooperation with the finance industry.

Applied probability This orientation is based on UE 1.4, 1.5, 1.6, 2.1, 2.2 and relies on the research group in probability and statistics of Laboratoire d'Analyse et de Mathématiques Appliquées, a common laboratory of University Paris-Est Marne-la-Vallée and University Paris-Est Créteil.

Analysis and applications (image, compressed sensing) Recommended for students with an interest in analysis, this program is based on UE 1.1, 1.2, 1.3, 2.4, 2.5 et 2.6. It focuses on research topics developed by scientists of the Universities Paris-Est Marne-la-Vallée, Paris-Est Créteil and Evry-Val-d'Essonne. It offers an initiation to recent techniques in analysis, with powerful applications in image analysis and signal processing.

Conditions of admission and applications

On line applications can be made on the web site `http://candidatures.univ-mlv.fr/`. If you need help for applying, please contact our secretaries (`florence.gamon@univ-mlv.fr` or `christiane.lafargue@univ-mlv.fr`, tel: 33 1 60 95 75 20).

Admitted students may register in any of the following institutions: university of Marne-la-Vallée, university of Paris XII-Val de Marne, university of Evry-Val d'Essonne.

Pedagogical organization

The second year consists of two semesters. Courses start on **Monday September, 19$^{\text{th}}$ 2011**.

The first semester courses are fundamental courses, with some of them oriented towards probability and others towards analysis. The final exams of the first semester are in December or January.

The second semester is devoted to more specialized courses (from January to March) and to a research training period (from April to June, with possible extension until September). The list of courses is subject to modifications during the first semester. Each course is credited with 9 ECTS[1].

The research training period may take place in an academic environment or in a research center in public or private industry. This research period is credited with 15 ECTS.

[1] 9 credits. The letters ECTS refer to European Credit Transfer System

Conditions for degree delivery

In the *finance* program, the students must pass UE *Tronc commun finance* (24 ECTS) and UE *Mathématiques financières approfondies* (21 ECTS), with a combined average of at least ten (out of 20), and they must achieve an internship of at least three months (credited with 15 ECTS).

The other orientations are made of 5 UE with 9 ECTS each, together with a research training period of at least three (credited with 15 ECTS). Each 9 ECTS UE consist of a thirty hour course. Registered students may, upon the agreement of the director of the program, and with a limit of up to 30 hours or 9 ECTS, take courses in other master programs. In order to obtain the degree, a student must achieve an average of at least 10 (out of 20) on its five best grades in the courses and a grade of at least 10 on the training period.

Career prospects

Students graduating from this program may apply for positions in industry (especially those majoring in finance or applied probability). Students with high records may be admitted in a doctoral program. PhD theses may be prepared in one of the research groups in connection with the master program:

laboratoire d'analyse et de mathématiques appliquées (UMR 8050 CNRS), Universities Paris-Est Marne-la-Vallée and Paris-Est Créteil,

équipe d'analyse et probabilités (EA 2172), University of Evry,

CERMICS, Centre d'Enseignement et de Recherche en Mathématiques, Informatique et Calcul Scientifique, Ecole Nationale des Ponts et Chaussées,

group "mathématiques et applications" , Ecole Supérieure d'Ingénieurs en Electrotechnique et Electronique.

Doctoral students may apply for various forms of financial aids: *allocations de recherche* from the Ministry for Education and Research, *bourses C.I.F.R.E.*, *bourses de l'École Nationale des Ponts et Chaussées*, ...). The *allocations de recherche* from the Ministry for Education and Research are allocated by *Doctoral Schools*.

List of the UEs (see detailed description in the following pages)

Finance orientation

F1 Basic courses in finance (Tronc commun finance, 24 ECTS)

- Stochastic calculus and applications in finance
- Monte-Carlo methods in finance
- Interest rate models
- Computer programing
- One week seminar on Quantitative Finance

F2 Advanced courses in mathematical finance (Mathématiques financières approfondies 21 ECTS)

- two courses with 6 ECTS to be taken out of the following list: *Treatment of market data: statistical aspects and calibration*, *Credit risk, counterparty risk*, *Risk measures in finance*, *Processes with jumps and applications to energy markets.*
- another 9 ECTS course (other than 1.4, 1.6 or 2.2).

Other UEs (9 ECTS courses)

1.1 Evolution equations: theory and algorithms

1.2 Techniques of analysis in partial differential equations

1.3 Analysis for variational models

1.4 Stochastic calculus and applications in finance

1.5 Statistics of discrete time processes

1.6 Stochastic Processes 2

2.1 Stochastic models

2.2 Modeling and simulation

2.3 Introduction to Malliavin calculus and its numerical applications in finance

2.4 Multifractal analysis, signal processing and image analysis

2.5 Introduction to non-linear dispersive PDEs

2.6 Asymptotic geometry, harmonic analysis and compressed sensing

F1 Tronc commun finance

This *Unité d'Enseignement* is organized jointly with the financial engineering program of Ecole des Ponts. The course *Stochastic calculus and applications in finance* is given at the university (see page **??** for an outline of the course).

The courses *Monte-Carlo methods in finance* and *Interest rate models* are given at École des Ponts. The course *Monte-Carlo methods in finance* presents simulation techniques of random variables for the effective computation of quantities of interest in finance (see page 6). Tutorial session in C++ programming are given in complement to this course. The course *interest rate models* presents various approaches to the modeling of interest rates and the computation of bond prices and interest rate derivatives (see page 7).

The *Seminar on "quantitative finance"*, organized by École des Ponts, offers the students an initiation to derivative markets, in particular with presentations by practitioners.

F2 Mathématiques financières approfondies

This compulsory UE of the finance program consists of one 9 ECTS course (to be selected from the list of 9 ECTS course, in agreement with the director of the program) and two 6 ECTS courses to be taken among the following:

- Treatment of market data and calibration (see page 8).
- Credit risk, counterparty risk (see page 9).
- Risk measures in finance (see page 10).
- Processes with jumps and applications to energy markets (see page 11).

Among 9 ECTS courses, the following are especially recommended: *Equations d'évolution : théorie et algorithmes* and *Introduction au calcul de Malliavin et applications numériques en finance.* The courses *Stochastic processes 2* and *Modélling and simulation* cannot be part of the finance orientation.

F1.1 Monte-Carlo Methods in finance

Lecturers: Bernard Lapeyre, Benjamin Jourdain, Eric Benhamou.

Part I : Monte-Carlo methods for the computation of integrals in $\mathbf{R}^n$

1. The Monte-Carlo method, introduction to variance reduction.
2. Variance reduction methods: control variates, importance sampling, stratification, conditioning, ...
3. Sequences with low discrepancy, theoretical aspects, classical examples (Halton, Faure, Sobol, Niederreiter, ...).
4. Using sequences with low discrepancy and quantization techniques in finance.
5. Introduction to American Monte Carlo. Description of the Longstaff-Schwartz algorithm and quantization.

Part II: Monte-Carlo methods for financial processes

In this part, we focus on the computation of option prices written as expectations of functionals of the stock price process. The Monte Carlo error can be decomposed into a bias term corresponding to the time discretization of the diffusion plus a statistical error. The bias term will be examined, before a review of variance reduction methods, followed by a discussion of the discretization of models with jumps:

1. Discretization of diffusions: classical schemes (Euler, Milshtein, ...), rates of convergence. Extrapolation techniques. Higher order schemes.
2. Discretization techniques for exotic options (barrier and look-back options, Asian options, ...).
3. Variance reduction for option pricing in diffusion models.
4. Simulation of models with jumps.

Part III: Monte Carlo methods viewed by a practitioner

1. Pricing of a complex path dependent interest rate derivative. Description of financial cash flows, risk to incorporate in the modeling, a first Monte Carlo computation for the convexity adjustment.
2. Computation of the TARN condition of the product. Comparison Monte Carlo vs Quasi Monte Carlo. Estimating implicit strikes. Discussion about calibration.

References:

- Paul Glasserman. *Monte Carlo methods in financial engineering*, volume 53 of *Applications of Mathematics (New York)*. Springer-Verlag, New York, 2004.
- Bernard Lapeyre, Etienne Pardoux, et Rémi Sentis. *Méthodes de Monte-Carlo pour les équations de transport et de diffusion*, volume 29 de *Mathématiques & Applications (Berlin)*. Springer-Verlag, Berlin, 1998.

F1.2 Interest rate models

Lecturer: Vlad Bally.
Purpose of the course

The purpose of the course is to present an introduction to commonly used models in the theory of interest rates. Three classes of models have emerged. The oldest viewpoint relates the behavior interest rates to the short rate. A number of models for the dynamics of short rates have been proposed, in connection with calibration purposes. However, short rate models fail to explain the evolution of zero coupon bonds in full generality. A new generation of models appeared: first, the Heath-Jarrow-Merton (HJM) model, based on *forward rates.* Then, there are the so called "market models" - the Brace-Gatarek-Musiela (BGM) model, and also Jamishdian's model - which focus on some specific derivatives and brings forward a modeling which allows the pricing of these products in closed form.

Outline of the course

Part 1. Short rate models.
a. General presentation: zero coupon bonds, short rates, forward rates.
b. The structure equation. PDE approach and martingale approach.
c. Classical short rate models: Vasicek, Ho and Lee, Hull and White, Cox-Ingersol-Ross.
d. Multi-factor models.
e. Affine models.

Part 2. Heath-Jarrow-Merton model (HJM).
a. Martingale modeling and the HJM drift condition.
b. Change of numéraire and forward probability measures.
c. Black formula.
d. Pricing of typical derivatives: caps, floors, swaps and swaptions. Swap rate.

Partie 3. Market models. The Brace-Gatarek-Musiela model (BGM).

References:

- Björk T. (1998), *Arbitrage Theory in Continuous Time*, Oxford University Press.
- Björk T.(1997), *Interest Rate Theory*, in Runggaldier (ed.) *Financial Mathematics,* Springer Lecture Notes in Mathematics **1656**. Springer Verlag, Berlin.
- Brigo D. et Mercurio F., *Interest rate models, theory and practice*, Springer Finance, 1998.
- Lamberton D., Lapeyre B. (1997), *Introduction au Calcul Stochastique Appliqué à la Finance*, 2nde édition, Ellipses.

F2.1 Treatment of market data: statistical aspects and calibration

Lecturers: Aurélien Alfonsi, Stéphane Crépey, Arnaud Gloter.

Statistical estimation of stochastic processes used in finance (diffusions, stochastic volatility models, GARCH models). Volatility estimation with high frequency data and microstructure noise.

General presentation of calibration issues and differences with statistical approaches. Introduction to ill-posed inverse problems and Tikhonov regularization. Reminder on optimization. Calibration of affine models and advantage of pricing by FFT in this context. Study of the Heston model. Calibration of the local volatility model and stability problems of Dupire's formula. Limits of calibration to marginal distributions only and Monte-Carlo calibration.

References:

- D. Dacunha-Castelle, M. Duflo. *Problèmes à temps mobiles*, Masson 1983.
- D. Bosq, H.T Nguyen. *A course in stochastic processes: Stochastic models and Statistical inference.*
- I. Basawa, P. Rao.*Statistical inference for stochastic processes.* Kluwer, 1996.

F2.2 Credit risk, counterparty risk

Lecturers: Stéphane Crépey, Monique Jeanblanc.

The world of credit risk has been evolving very rapidly in the last few years. Before the crisis, the main issue was the modelling of credit derivatives. The crisis put the focus on the fundamental source of credit risk, namely counterparty risk. This risk, which occurs in any OTC transaction, is the risk of non payment of the expected cash-flows due to the default of one of the parties. An even more recent evolution stems from systemic counterparty risk. Through its impact on discounting factors, this systemic feature of risk has consequences for all derivative markets.

The first part of the course will be devoted to mathematical tools that can be used for modelling events related to default: jump processes, enlargement of filtrations. The second part will cover counterparty risk in its various aspects: counterparty risk for non credit derivatives, counterparty risk for credit derivatives, systemic counterparty risk, interactions between counterparty risk and funding.

References:

- Bielecki, T.R. and M. Rutkowski (2002) *Credit Risk: Modelling, Valuation and Hedging.* Springer-Verlag.
- Cesari, G., Aquilina, J. and Charpillon, N. (2010). *Modelling, Pricing, and Hedging Counterparty Credit Exposure.* Springer Finance.
- Cont, R. and Tankov, P. (2004) *Financial Modeling with Jump Processes.* Chapman & Hall/CRC.
- Mansuy, R. and Yor, M. (2006), *Random Times and (Enlargement of) Filtrations in a Brownian Setting.* Springer, Lectures Notes in Mathematics, 1873. (Chapitre 1).
- Schnbucher P.J. (2003), *Credit Derivatives Pricing Models.* Wiley.

F2.3 Risk measures in finance

Lecturers: Aurélien Alfonsi (École des Ponts), Peter Tankov (École Polytechnique).

The mastery of risk is at the heart of the bank industry's preoccupations, as testified by the Basel Committee's recommendation (national convergence of measure norms on reserve funds). The implementation of these recommendations yields the hiring of specialist in risk control services of banks. The purpose of this course is to present in a theoretical part the tools for measuring risks. The main topics are: monetary risk measures and the representation of convex risk measures, extreme value theory and the multidimensional representation of risks via copulas. In a second practical part, practitioners from Société Générale will present methods by various departments in order to assess financial risk.

The detailed contents can be found on the site

`http://cermics.enpc.fr/~alfonsi/mrf.html`.

This course is financed by the "Chaire Risques Financiers" of Société Générale, Ecole Polytechnique and Ecole des Ponts.

References:

- Basel Committee on Banking supervision. *International convergence of capital measurement and capital standards.*
- Föllmer H. and A. Schied (2004) *Stochastic finance. An introduction in discrete time.* De Gruyter Studies in Mathematics **27**, 2004.
- McNeil A.J., R. Frey and P. Embrechts *Quantitative risk management. Concepts, techniques and tools.* Princeton Series in Finance, 2005.
- Roncalli T. *La gestion des risques financiers.* Economica. 2004.

F2.4 Processes wit jumps and applications to energy markets

Lectures: Jean-François Delmas, Benjamin Jourdain.

The purpose of this course is to introduce Lévy processes and stochastic calculus with jumps wit a view to applications to energy markets. Lévy processes are processes with with independent and stationary increments which generalize Brownian motion by relaxing the continuity assumption on the sample paths. After showing, through the Lévy-Kyntchine formula, that the law of a Lévy process depends only on its so-called characteristic triplet, we will give a representation of Lévy processes in terms of Brownian motion and a Poisson point measure. We will then construct stochastic integrals with respect to the Poisson measure and to the compensated Poisson measure and will prove Itô's formula in this context. We will also cover some elements of stochastic calculus for general semimartingales. We will then present applications of processes with jumps in mathematical finance and particularly for markets of energy commodities: organization of the market, derivative products, pricing with Lévy processes, numerical methods and application to gas storage derivatives.

Some of the sessions will involve lecturers from *Électricité de France*. This course is financed by the "Chaire Risques Financiers" from the *Fondation du Risque*.

This course is taught at Ecole des Ponts.

1.1 Evolution equations: theory and algorithms

Lecturer: Robert Eymard.

The aim of this course is to study the approximation of non linear evolution problems, especially those related to variational inequalities and degenerate parabolic equations. These problems are used in mathematical finance, in particular for American option pricing.

We start with the discretization of stationary problems (in particular, finite element methods will be briefly reviewed). Then, the case of linear evolution (heat equation) is studied and, finally, parabolic degenerate equations are studied and the link with variational inequalities is established.

- **Theoretical concepts covered**.
  Sobolev spaces are used, as well as the theorems of Riesz, Lax-Milgram, and Riesz-Fréchet-Kolmogorov, for the convergence of approximate solutions to the exact solution.

- **Numerical study**.
  The theme of the course being the applications to mathematical finance, the students are invited to implement the computation of the price of an American option using the methods of the course. This work is take into account in the final grade.

References:

- Y. Achdou, O. Pironneau, *Computational methods for option pricing*, SIAM series: Frontiers in Applied Mathematics, 2005.
- H. Brezis, *Analyse Fonctionnelle*, Masson, 1983.
- P.G. Ciarlet , J.L.Lions, *Handbook of Numerical Analysis*, volume 1, North Holland, 1992.
- R. Dautray, J.L. Lions, *Analyse Mathématique et Calcul Numérique pour les Sciences et Techniques*, Masson, (chapitres XIV, XV, XVIII et XX).
- P. A. Raviart, J. M. Thomas, *Introduction à l'Analyse Numérique des EDP*. Masson, 1983.

Prerequisites: Integration theory, notions of functional analysis.

1.2 Techniques of analysis in partial differential equations

Lecturer: Marco Cannone.

The purpose of this course is to complete the students' knowledge in classical analysis and introduce some material in functional and harmonic analysis. We give applications to partial differential equations and to multifractal analysis.

The following topics are covered:

- Banach spaces: duality and weak topology
- Analysis in Lebesgue spaces: interpolation and applications
- Spaces of distributions: approximation, regularization and Fourier analysis
- Sobolev spaces: characterization, Sobolev embedding, Rellich compactness theorem. Applications of Sobolev spaces to Partial Differential Equations. Variational methods. Application to the Dirichlet problem, maximum principle.
- Introduction to Wavelets: construction and examples of bases, characterization of function spaces (Besov and Sobolev spaces), applications

References:

- R.A. Adams, Sobolev Spaces, Academic Press, 1975.
- H. Brezis, Analyse fonctionnelle, Masson, 1983.
- F. Hirsch, G. Lacombe, Eléments d'analyse fonctionnelle, Masson, 1997.
- Y. Meyer, Ondelettes et opérateurs, tome 1, Hermann, 1990
- Y. Meyer, Ondelettes et algorithmes concurrents, Hermann, 1993
- W. Rudin, Analyse fonctionnelle, Ediscience.
- K. Yosida, Functional Analysis, Springer-Verlag, Sixth edition, 1995.

1.3 Analysis for variational models

Lecturers: Etienne Sandier.

This course deals with problems stated in variational form, i.e. in the form of an energy to be minimized, and with the relevant techniques of nonlinear analysis. The following list gives an idea of covered topics:

- Basic concepts: direct method, Euler-Lagrange equations.
- Examples with a lack of compactness. Plateau's problem for soap bubbles or gauge invariant problems. Examples where the minimum is not attained.
- Properties related to the presence of symmetries: conservative form of the equations, the principle of symmetric critical points, rearrangement and symmetrization.
- Convex functional and Fenchel's transform: formulations of the obstacle problem, Kantorovitch duality for optimal transportation.
- Variational convergence, or Gamma-convergence, example of Modica-Mortola, Ginzburg-Landau functional.

References:

- Brezis, H., *Analyse fonctionnelle*, Masson.
- Evans, L. C., *Weak convergence methods for non linear partial differential equations*, Expository lectures held at Loyola, University of Chicago, June 27-July 1, 1988, CBMS, vol. 74 (American Mathematical Society, 1990).
- Struwe, M., *Variational methods*, Springer (1990); second edition: Ergebnisse Math. 34, Springer (1996); third edition (2000).

1.4 Stochastic calculus and applications in finance

Lecturer: Damien Lamberton.

The purpose of this course is to present basic examples of stochastic processes in continuous time and to give the students access to applications in various fields: finance, reliability, Monte-Carlo methods, partial differential equations.

- Brownian motion: construction, path properties.
- Martingales in continuous time, optional sampling theorem.
- Stochastic integration, Ito's formula. Application to finance (Black and Scholes'model).
- Stochastic differential equations. Connection with partial differential equations.
- Applications to option pricing. Variational inequalities and American options.

References:

- N. Bouleau, *Processus Stochastiques et Applications*, Hermann (1988).
- F. Comets, M. Meyre, *Calcul stochastique et modèles de diffusions*, Dunod (2006).
- J. Hull, *Options, futures and other derivatives*, Prentice Hall (2006).
- I. Karatzas, S. Shreve, *Brownian motion and Stochastic Calculus*, Springer-Verlag (1987).
- D. Lamberton, B. Lapeyre, *Introduction au Calcul Stochastique Appliqué à la Finance*, 2nde édition, Ellipses (1997).
- R. Portait, P. Poncet *Finance de marché*, 2nde édition, Dalloz (2009). Springer (1997).
- D. Revuz, M. Yor, *Continuous martingales and Brownian motion*, Springer-Verlag (1991).

1.5 Statistics of discrete time processes

Lecturers: Cristina Butucea, Florence Merlevède

1st part. Concerning many phenomena, observations are not independent but they rather have the following property: the observations of the "past" and of the "present" may have a big influence on the near future observations, and a decreasing influence on the long run observations. For the statistical analysis of such phenomena,it is natural to use random sequences for which the dependence decreases with time. The main limit theorems that will be presented deal with the convergence of the empirical process (which describes observations up to time t) towards a Brownian bridge. This convergence will be studied in the case of non iid random variables, natural dependence coefficients in this context will be introduced and classes of processes satisfying the dependence conditions will be presented. These dependence coefficients will also be used in the study of the integrated $\mathbf{L}_p$ ($p \geq 2$) error of kernel estimators of the density of stationary processes.

2nd part. We will investigate the estimation of functions f (probability density, spectral density...) and functionals (the energy $\int f^2$, the excess mass $\int (f - \lambda)_+$ for a threshold $\lambda > 0$) in connection with stationary processes. Other estimation techniques will be introduced: estimators by projection on an orthonormal basis (for example, Fourier series, wavelets).

More specifically, in the case of sequences of independent identically distributed random variables, we will deal with inverse problems. These are situations where some transform of the function of interest can be estimated from the observations. The estimation methods have a slower rate of convergence than for the direct problem. We will try to formalize the loss in performance and the transform intrinsic to the problem.

References:

- Billingsley, P. (1999). Convergence of probability measures. Second edition. *John Wiley & Sons, New York.*
- Dedecker, J, Doukhan P., Lang, G., León J.R., Louhichi, S. and Prieur, C. (2007). Weak dependence with examples and applications. *Lecture notes in Statistics.* **190** Springer.
- Hall, P. et Heyde, C. C. (1980). Martingale Limit Theory and its Applications. *Lecture notes in Statistics.* **129** *Academic Press, New York-London.*
- Härdle, W., Kerkyacharian, G., Picard, D. and Tsybakov, A. (1998). Wavelets, approximation, and statistical applications. *Springer-Verlag, New York.*
- Tsybakov, A. B. (2004). Introduction à l'estimation non-paramétrique. *Springer-Verlag, Berlin.*

1.6 Stochastic Processes 2

Lecturer: Djalil Chafaï

This course is a UE from the professionnal master *IMIS* (Ingénierie Mathématique, Informatique et Statistique), with a specific grading for students of the reseach master program. It involves a project uner Scilab, Octave, or R. The focus is put on simulation and examples. Main topics:

- Discrete time Markov chains. Recursive approach.
- Limit theorems, coupling, simulation, MCMC methods.
- Poisson process and continuous time Markov chains.
- Queueing theory, Yule process. Semi-Markovian extensions.

2.1 Stochastic models

Lecturer : Djalil Chafaï.

The aim of this course is to contribute to the construction of a classical probabilistic cuture, with an eclectic view, based on models, problems, tools in action. Each of the ten three hour sessions is devoted to the study of a specific stochastic model. Some sessions start with fifteen minute presentation by one of the students on some briefly covered topic of the previous sessions. The choice of the models may vary each year.

Prerequisites : M1 level in mathematics, particularly in probability.

Lecture notes (about 90 pages) : http://djalil.chafai.net/enseignement.html

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2.2 Modelling and simulation

Lecturer: Thierry Jeantheau.

This course is a UE of the professional master *IMIS* (Ingénierie Mathématique, Informatique et Statistique), with a specific grading for students of the research program.

It concerns students with some background in probability, especially Markov chains. Various methods for the simulation of random variables are presented. The case of random vectors is also treated, and the concept of copula is introduced for the modelling and simulation of specific dependence structures. Monte Carlo methods for the computation of integrals are presented. The use of Markov chains for the simulation of some complicated distributions (MCMC methods) is discussed, especially the Metropolis algorithm. We also show how Monte Carlo methods can be used for optimization problems, by presenting the simulated annealing algorithm.

All the methods covered by this course are also implemented by the students, using some scientific (Scilab) or statistical (R) software.

1. Simulation of random variables and vectors.
2. Introduction to modelling by Copulas and simulation.
3. Monte Carlo methods, application to the computation of integrals.
4. Simulation using Markov chains (MCMC method), Metropolis algorithm.
5. Application to optimization, simulated annealing algorithm.

2.3 Introduction to Malliavin calculus and its numerical applications in finance

Lecturer: V. Bally

The aim of this course is to give an elementary introduction to Malliavin Calculus and its applications. We especially focus on the numerical applications in mathematical finance. Students are supposed to have a good knowledge of basic stochastic calculus. The main points in the course are the following:

- **General presentation.** Abstract integration by parts formula and applications. Differential operators, duality formula. Clark-Ocone representation formula and hedging strategies. Applications to diffusions.
- **Computation of sensitivities**. The *greeks*.
- **American options** Computation of conditional expectations. Dynamic programming and Monte Carlo methods. Localization, variance reduction.
- **Sobolev spaces on Wiener space.**
- **Chaos decomposition.**

References:

- D Nualart. The Malliavin calculus and related topics. Springer Verlag, 1995
- N. Ikeda and S. Watanabe: Stochastic Differential Equations and Diffusion Processes. North Holland 1989.
- S. Watanabe: Lectures on Stochastic Differential Equations and Malliavin calculus. Tata Institute of Fundamental Research, Springer Verlag 1984.

2.4 Multifractal analysis, signal processing and image analysis

Lecturers: Stéphane Jaffard, Stéphane Seuret.

Multifractal analysis was first introduced in physics and in signal processing in order to study and model first turbulent flows, then various signals (in traffic, physiology...). Relationships with PDEs and number theory recently emerged. Wavelets allow for fast implementation of these methods in signal and image processing where these concepts give new simulation and classification tools

The purpose of the course is to introduce the fundamental concepts used in multifractal analysis, and then to focus on one area of applications. The course will be split into two parts:

1. Concepts and fundamental results
   - Hausdorff Dimension (definition and techniques of computation).
   - Sobolev spaces, Besov spaces, oscillation; Hölder exponent
   - wavelet bases.
   - Definition and upper bounds for singularity spectra.
   - Multifractal formalism. Applications in signal processing and image analysis.
2. Applications will be chosen in the following list :
   - Statistical methods for the estimation of singularity spectra of signals and images.
   - Turbulence models (multiplicative cascades).
   - Random wavelet series.
   - Multifractal analysis of remarkable functions: Bolzano, Riemann and Polya functions, Davenport series
   - Introduction to ubiquity methods..
   - Multifractal analysis of Lévy processes, application to Burgers' equation.
   - Wavelet analysis of fractals domains

Lecture notes will be handed in to the students.

2.5 Introduction to nonlinear dispersive PDEs

Lecturers: Valeria Banica, Pierre-Gilles Lemarié-Rieusset

This course is divided into two parts. The first part is devoted to the Schrödinger equation, which appears in quantum field theory, nonlinear optics and fluid mechanics. The second part deals with semi-linear wave equations.

In both parts, we first study some properties of the associated linear equation using techniques from harmonic analysis. We then carry on to highlight some qualitative properties in the semi-linear case: local existence, explosion in finite time, global existence, asymptotic behavior...

References:

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- J. Ginibre, *Introduction aux équations de Scrödinger non linéaires.*, Editions de l'Université de Paris-Sud (1995).
- P.-G. Lemarié-Rieusset: *Recent Developments in the Navier-Stokes Problem*, Research Notes in Mathematics, Chapman & Hall/CRC, (2002).
- C. Sogge, *Lectures on nonlinear wave equations*, monographs in analysis II, International Press, Cambridge (1995).
- E.M. Stein, G. Weiss, *Introduction to Fourier Analysis on Euclidean Spaces*, Princeton University Press (1971).
- C. Sulem and P.-L. Sulem, *The nonlinear Schrödinger equation. Self-focusing and wave collapse*, Applied Mathematical Sciences. 139. New York, NY : Springer. xvi, 350 p. (1999).
- W. A. Strauss, *Nonlinear wave equations*, CBMS Regional Conference Series in Mathematics, **73**, AMS, Providence (1989).
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2.6 Asymptotic geometry, harmonic analysis and compressed sensing

Lecturers: Matthieu Fradelizi, Olivier Guédon.

At the interface between classical geometry, local theory of Banach spaces and probability theory, asymptotic geometry describes phenomena which occur in a measured metric space with dimension n as n goes to infinity. The purpose of the course is to present some important results relating measure concentration phenomena and harmonic analysis.

The study of isoperimetric inequalities and their functional equivalent on the sphere, the Euclidean space, the Gaussian space and the Walsh space of dimension n leads, in each of these areas, to phenomena of measure concentration which have implications in areas as varied as the geometry of convex bodies in high dimension, graph theory and the study of singular values ??of random matrices. Using concentration inequalities, one can establish universal properties of sections of convex bodies. Moreover, the reconstruction of signals with small support algorithm by means of an ℓ_1-minimization algorithm is intimately connected to the study of Euclidean sections of the ℓ_1 ball.

The course will consist of two parts. In the first part, we introduce semi-group methods and their applications in convex geometry, and in discrete or Gaussian geometry. The Ornstein-Uhlenbeck and heat semi-groups will be particularly studied. We will prove the inequalities of Brunn-Minkowski and Ehrhardt and their functional forms. The isoperimetric inequality for different measures (Lebesgue, Gaussian, Bernoulli) will be deduced, as well as functional inequalities of various types (Poincar, Sobolev, log-Sobolev inequality) and finally measure concentration inequalities in each of these spaces. Finally, we will present Ledoux' very recent proof of a theorem of E.. Milman, stating that, in a context of positive curvature, isoperimetry can be deduced from concentration.

The second part will be devoted to the study of classical inequalities of harmonic analysis and of the geometry of convex bodies. We present a geometric version of the Brascamp-Lieb inequalities. These inequalities are a refinement of the inequalities of Young about the L_r-norm of the convolution product of two functions f and g in L_p and L_q respectively, with $1/p+1/q = 1/r$. In a second part, we will present the results of convex geometry in high dimension. Indeed, in this situation, convex bodies have universal properties and we illustrate this phenomenon by presenting the Dvoretzky theorem: for any symmetric convex set in $\mathbf{R}^n$, there are sections by subspaces of dimension $\log n$ which are almost Euclidean. If there are enough students interested, we will also study the recently developed theory on the exact or approximate reconstruction vectors with small support.